

# README file for "Angularly resolved Atomic Time Delays"

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## 1 Time-Dependent Schrödinger Equation (TDSE) calculations

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### 1.1 Files

1. [Angular\\_delays\\_TDSE\\_0\\_4\\_0\\_0\\_180\\_2.dat](#)
2. [Angular\\_delays\\_TDSE\\_20\\_4\\_0\\_0\\_180\\_2.dat](#)
3. [Angular\\_delays\\_TDSE\\_40\\_4\\_0\\_0\\_180\\_2.dat](#)
4. [Angular\\_delays\\_TDSE\\_60\\_4\\_0\\_0\\_180\\_2.dat](#)
5. [Angular\\_delays\\_TDSE\\_80\\_4\\_0\\_0\\_180\\_2.dat](#)
6. [Angular\\_probs\\_TDSE\\_4\\_0\\_0\\_60\\_20\\_0\\_180\\_2.dat](#)

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## 1.2 Description

Files with name pattern *Angular\_delays\_TDSE- $\Theta$ -4-0-0-180-2.dat* contain the angularly resolved atomic time delays  $\tau_{at}(\theta)$  we obtain from TDSE simulations of RABBITT spectra for hydrogen atoms initially in the  $1s$  ground state. Each file contains the results for the relative polarization angle  $\Theta = 0^\circ, 20^\circ, 40^\circ, 60^\circ,$  and  $80^\circ$  between the polarization vectors of pump and probe pulses. The first column indicates the photoelectron emission angle in the  $xy$ -plane measured from the positive  $x$ -axis, the polarization direction of the attosecond pulse train (see Fig. 1 in Ref. [1]). The following columns in the data file contain the angularly resolved time delays  $\tau_{at}(\theta)$  for SBs 12 to 26.

The file with name [Angular\\_probs\\_TDSE\\_4\\_0\\_0\\_60\\_20\\_0\\_180\\_2.dat](#) contains the results for the fitting parameter  $A$  for SBs 12 to 26, as a function of the electron emission angle and for each relative polarization angle  $\Theta$ . Again, we obtain these results from TDSE simulations of RABBITT spectra for hydrogen atoms initially in the  $1s$  ground state.

## 1.3 Method

Utilizing the Qprop code [2], we numerically solve the TDSE for each angle  $\Theta$  and 41 different values of the delay  $\tau$  between the attosecond pulse train and the infrared (IR) laser. These delays span one IR period, and allow us to find the dependence of angle-resolved RABBITT spectra on the delay  $\tau$ . Then, we fit the signal for each sideband with the general expression  $I_{2q} = A + B \cos(2\omega_0\tau - \phi_{at})$ , from which we obtain the angular dependence of the parameters  $A$  and  $B$ , and the atomic time delay  $\tau_{at} = \phi_{at}/2\omega_0$ .

The relevant parameters to reproduce the TDSE simulations are the following (values in atomic units unless otherwise stated):

- Radial grid step: 0.24
- Cutoff radius: 850
- T-surff radius: 1700
- Number of angular momenta in expansion: 4
- Time step for propagation: 0.06
- IR intensity: 0.05 TW/cm<sup>2</sup>
- IR angular freq.  $\omega_0$ : 0.056954190
- IR pulse duration: 10 IR-cycles
- XUV pulse duration: 3 IR-cycles
- XUV Harmonic orders: 11 to 27

## 1.4 Observations

The relative time delays  $\Delta\tau_{at}(\theta)$  in Figs. (2) and (3) from Ref. [1], are obtained by taking the difference  $\tau_{at}(\theta) - \tau_{at}(0^\circ)$  from the datasets above.

## 2 Second-order perturbation theory (SOPT) calculations

Author(s): D.I.R. Boll and L. Martini

### 2.1 Files

1. [Angular\\_delays\\_SOPT\\_0\\_0\\_180\\_2.dat](#)
2. [Angular\\_delays\\_SOPT\\_20\\_0\\_180\\_2.dat](#)
3. [Angular\\_delays\\_SOPT\\_40\\_0\\_180\\_2.dat](#)
4. [Angular\\_delays\\_SOPT\\_60\\_0\\_180\\_2.dat](#)
5. [Angular\\_delays\\_SOPT\\_80\\_0\\_180\\_2.dat](#)

### 2.2 Description

Files with name pattern *Angular\_delays\_SOPT\_Θ\_0\_180\_2.dat* contain the angularly resolved atomic time delays  $\tau_{at}(\theta)$  we obtain from SOPT calculations for hydrogen atoms initially in the  $1s$  ground state. Each file contains the results for the relative polarization angle  $\Theta = 0^\circ, 20^\circ, 40^\circ, 60^\circ,$  and  $80^\circ$  between the polarization vectors of pump and probe pulses. The first column indicates the photoelectron emission angle in the  $xy$ -plane measured from the positive  $x$ -axis, the polarization direction of the attosecond pulse train (see Fig. 1 in Ref. [1]). The following columns in the data file contain the angularly resolved time delays  $\tau_{at}(\theta)$  for SBs 12 to 26.

### 2.3 Method

For initial atomic  $s$  states, in Ref. [1] we show that angularly resolved atomic phase imprinted on RABBITT sidebands is given by

$$\tan(\phi_{at}) = \frac{\sum_{L,L'} |T_L^+| |T_{L'}^-| g_{L,L'} \sin(\phi_{L'}^- - \phi_L^+)}{\sum_{L,L'} |T_L^+| |T_{L'}^-| g_{L,L'} \cos(\phi_{L'}^- - \phi_L^+)}, \quad (1)$$

where  $|T_L^\pm|$  and  $\phi_L^\pm$  are the modulus and phase of (pseudo) radial matrix elements contributing to final states with angular momentum  $L$ , from absorption (+) and emission (−) channels, respectively. The full angular dependence of atomic phase  $\phi_{at}$  is contained in the  $g_{L,L'}$  functions (see appendix in Ref. [1]). Therefore, to obtain the angularly-resolved atomic time delays from SOPT calculations we substitute into Eq. (1) the results for radial matrix elements  $T_L^\pm$  obtained from Second-order Perturbation Theory (SOPT), reported previously [3].

## 2.4 Observations

The relative time delays  $\Delta\tau_{at}(\theta)$  in Figs. (2) and (3) from Ref. [1], are obtained by taking the difference  $\tau_{at}(\theta) - \tau_{at}(0^\circ)$  from the datasets above.

# 3 Model calculations

Author(s): D.I.R. Boll

## 3.1 Files

1. [Angular\\_delays\\_Model\\_0\\_0\\_180\\_2.dat](#)
2. [Angular\\_delays\\_Model\\_20\\_0\\_180\\_2.dat](#)
3. [Angular\\_delays\\_Model\\_40\\_0\\_180\\_2.dat](#)
4. [Angular\\_delays\\_Model\\_60\\_0\\_180\\_2.dat](#)
5. [Angular\\_delays\\_Model\\_80\\_0\\_180\\_2.dat](#)

## 3.2 Description

Files with name pattern *Angular\_delays\_Model\_Θ\_0\_180\_2.dat* contain the angularly resolved atomic time delays  $\tau_{at}(\theta)$  we obtain from ACC-RME model [4] calculations for hydrogen atoms initially in the  $1s$  ground state. Each file contains the results for the relative polarization angle  $\Theta = 0^\circ, 20^\circ, 40^\circ, 60^\circ, \text{ and } 80^\circ$  between the polarization vectors of pump and probe pulses. The first column indicates the photoelectron emission angle in the  $xy$ -plane measured from the positive  $x$ -axis, the polarization direction of the attosecond pulse train (see Fig. 1 in Ref. [1]). The following columns in the data file contain the angularly resolved time delays  $\tau_{at}(\theta)$  for SBs 12 to 26.

## 3.3 Method

We obtain the angularly-resolved atomic time delays from model calculations by substituting into Eq. (1) the results for radial matrix elements  $T_L^\pm$  obtained from ACC-RME model [4], reported previously [3].

## 3.4 Observations

The relative time delays  $\Delta\tau_{at}(\theta)$  in Figs. (2) and (3) from Ref. [1], are obtained by taking the difference  $\tau_{at}(\theta) - \tau_{at}(0^\circ)$  from the datasets above.

## 4 Changelog

### References

- [1] D. I. R. Boll, L. Martini, and O. A. Fojón. Two-color polarization control on angularly resolved attosecond time delays. *Physical Review A*, submitted.
- [2] Vasily Tulsy and Dieter Bauer. Qprop with faster calculation of photoelectron spectra. *Computer Physics Communications*, 251:107098, 2020.
- [3] D.I.R. Boll, L. Martini, and O.A. Fojón. Two-photon two-color transition matrix amplitudes. Repositorio Institucional CONICET Digital. <http://hdl.handle.net/11336/156629>, 2022. Creation date: 2022/05/05.
- [4] D. I. R. Boll, L. Martini, and O. A. Fojón. Analytical model for attosecond time delays and fano's propensity rules in the continuum. *Phys. Rev. A*, 106:023116, Aug 2022.